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2. DESIGN OF HYDROCYCLONES

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Foreword

The Raw Material Research and Development Council (RMRDC) was set up in 1987 with the objective of promoting value-added transformation of Nigeria's raw material resources as a strategy for sustainable industrial development; a task which it has been engaged in for the past twenty two years. In its avowed desire to fulfil its mandate, the Council has been building local capacity in design and development of process equipment, but discovered a yawning gap in local availability of appropriate literature and computer software to aid in the design process.

To address these problems the Council set up a team of eminent scholars from different institutions of higher learning and the private sector to develop appropriate software for the design of process equipment. The experience gained in this regard led to the publication of the first RMRDC Monograph series on cyclone.

The second in this series is this book- Design of Hydrocyclones. If you are an equipment designer fabricator, student of higher institution you will find this book useful as it presents step by step guide on hydrocyclone design, relevant design equations and computer aided design algorithm.

ENGR. (PROF.) A. P. ONWUALU, FAS
Director-General/CEO
Raw Materials Research & Development Council.

Preface

Hydrocyclones were originally patented in 1891 as a device for desanding water supplies but are now widely used in many other areas. By harnessing the tremendous power of centrifugal force, hydrocyclones have become standard equipment for clarification of slurries (removing solids) and thickening (removing liquids), washing of solids, removing gases from liquid as well as separating immiscible liquids.

Having no moving parts hydrocyclones rely on the centrifugal force created by the incoming slurry feed stream to make solid-liquid or liquid-liquid separations based on particle specific gravity and particle size. To create the necessary force, the potential energy of a process slurry is converted to kinetic energy at the cyclones feed inlet.

Although design procedures for hydrocyclones are proprietary, design parameters are known to depend on body diameter and by defining a ‘standard’ hydrocyclone as that cyclone which has the proper geometrical relationship between the cyclone diameter, inlet area, vortex finder, apex orifice, and other parameters, it is possible to scale-up the dimensions of a ‘standard cyclone’ for a particular application. This monograph has, therefore, assembled and presented available information in the literature on hydrocyclones and programmed the information into a simplified software package.

In the monograph, the first chapter presents an overview of hydrocyclones including performance parameters while the next chapter discusses the design equations and parameters. The last two chapters dwell, respectively on computer-aided design of hydrocyclones and validation of the design software that was produced.

We gratefully acknowledge the contributions of Mrs I.O Ejuya and I. I. Ismail, the in-house members of the CAPED (computer–aided process engineering design) group within the Raw Materials Research and Development Council. They provided both the logistic and technical support that were necessary to get this monograph to its present form.

Our special thanks go to the Director General, Raw Materials Research and Development Council, Engr (Prof) A. P. Onwualu, for sponsoring the project. We also appreciate the tremendous support and encouragement of Engr. G. Ladan, Deputy Director, Machinery and Equipment Development Division, for infrastructural support.

Lastly, we also acknowledge the various contributions of other staff of the Council.

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1/ *Introduction*

1.1 OVERVIEW

Hydrocyclones have come a long way since an original patent was issued in 1891 for a device to de-sand water supplies. For example, hydrocyclones are widely used in precipitation and crystallisation operations to remove large product crystals, while returning finer seed crystals back to the process. Similarly, cyclones are used in grinding and crushing operations to produce a fine product overflow while returning the coarse material back to the grinding stage. An example of this application is the limestone grinding stage of flue gas desulfurizarion. Here the fine limestone from the cyclone overflow is used to remove sulphur dioxide from power plant flue gas.

Hydrocyclones can also be used to remove most of the solids from a process stream. For example, units are commonly used to de-sand river water to provide clean process water and protect downstream equipment. The hydrocyclones can be used in combination with, or as replacements for, filters or centrifuges, to lower overall capital and operating costs. The benefits are lower maintenance and reduced operator attention. Likewise, hydrocyclones have replaced mechanical classifiers in most grinding plants, as they are more efficient, especially in the finer size ranges, and require less floor space. Due to the relatively short residence time of particles within the cyclone, the mill circuit can rapidly be brought into balance if any changes are made and oxidation of particles within the circuit is reduced.

Separations based on particle specific gravity can also be achieved using hydrocyclones. The most common use is in coal processing operations, where lighter coal is removed from shale and other unwanted high-specific-gravity materials to produce a clean coal produce. In recycling applications, particle-specific-gravity separations are used with increasing frequency to remove metals, plastics and other reusable materials. The most common application is for separation of soft drink bottles made of different plastics.

Thus, harnessing the tremendous power of centrifugal force, they have become standard equipment for

- Clarification of slurries (removing solids) and thickening (removing liquids)
- Classification of solids
- Washing of solids
- Removing gases from liquid
- Removing immiscible liquid from liquid

Having no moving parts hydrocyclones rely on the centrifugal force created by the incoming slurry feedstream to make solid-liquid or liquid-liquid separations based on particle specific gravity and particle size. To create the necessary force, the potential energy of a process slurry is converted to kinetic energy at the cyclones feed inlet. The inlet configuration allows the slurry to begin its rotational motion inside the cyclone.

Higher-density and larger larger-sized particles have larger centrifugal forces associated with them and tend to migrate to the cyclone wall, down the length of the cyclone and out through the apex of the underflow. Lighter, smaller particles, tend to follow the fluid flow and are forced into the vortex of the cyclone and out through the overflow. Hydrocyclone operation can be on stand-alone basis or in combination with thickeners, clarifiers and strainers.

When hydrocyclones are unable to meet the desired operation in one stage, they can be used in sequence with another stage of hydrocyclones or other types of process equipment. Hydrocyclones are frequently used as protection or pre-treatment devices to improve the performance or decrease the cost of other slurry system equipment, such as pumps, valves, centrifuges, filters and screens.

1.11 Description and Basic Operation

A typical hydrocyclone (Figure 1.1) consists of a conically shaped vessel, open at its apex, or underflow, joined to a cylindrical section, which has a tangential feed inlet. The top of the cylindrical section is closed with a plate through which passes an axially mounted overflow pipe. The pipe is extended into the body of the cyclone by a short, removable section known as the *vortex finder*, which prevents short circuiting of feed directly into the overflow.

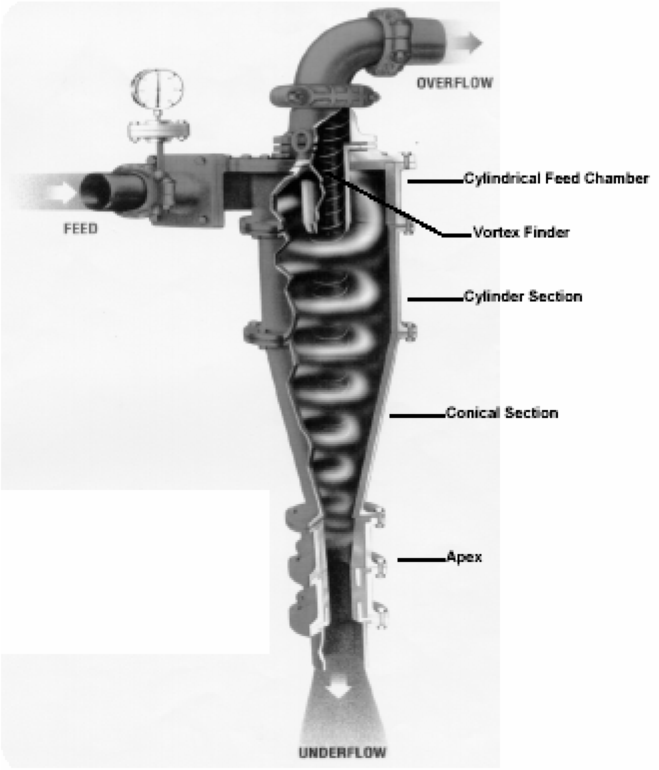

Figure 1.1. The Hydrocyclone

The feed is introduced under pressure through the tangential entry which imparts a swirling motion to the pulp. This generates a vortex in the cyclone, with a low pressure zone along the vertical axis. An air core develops along the axis, normally connected to the atmosphere through the apex opening, but in part created by dissolved air coming out of solution in the zone of low pressure.

Figure 1.2 shows the nomenclature that is used here to discuss hydrocyclones. The sketch shows the fundamental configuration around which modern hydro cyclones have evolved. The cyclone is divided into two major parts, the cylindrical section and the conical section. The cyclone diameter is the dimension which is referred to when specifying a cyclone size. The cylinder length is simply the length of the straight walls above the cone. Within the cylinder, we find the inlet orifice, the overflow orifice, and the vortex finder. The cone is the portion of the cyclone converging from the cylinder to the underflow orifice. The cone is primarily characterized in two ways; the cone length and the cone angle. Typical designs find optimum performance with cone lengths 4-5 times the cylinder diameter. As a gross generalization, the longer the cone, the more efficient the hydro cyclone.

The classical theory of hydro-cyclone action is that particles within the flow pattern are subjected to two opposing forces – an outward centrifugal force and an inwardly acting drag. The centrifugal force developed accelerates the settling rate of the particles (there is evidence to show that Stoke's law applies with reasonable accuracy to separations in cyclones of conventional design), thereby separating particles according to size and specific gravity. Faster settling particles move to the wall of the cyclone, where the velocity is lowest, and migrate to the apex opening. Due to the action of the drag force, the slower settling particles move towards the zone of low pressure along the axis and are carried upward through the vortex-finder to the overflow.

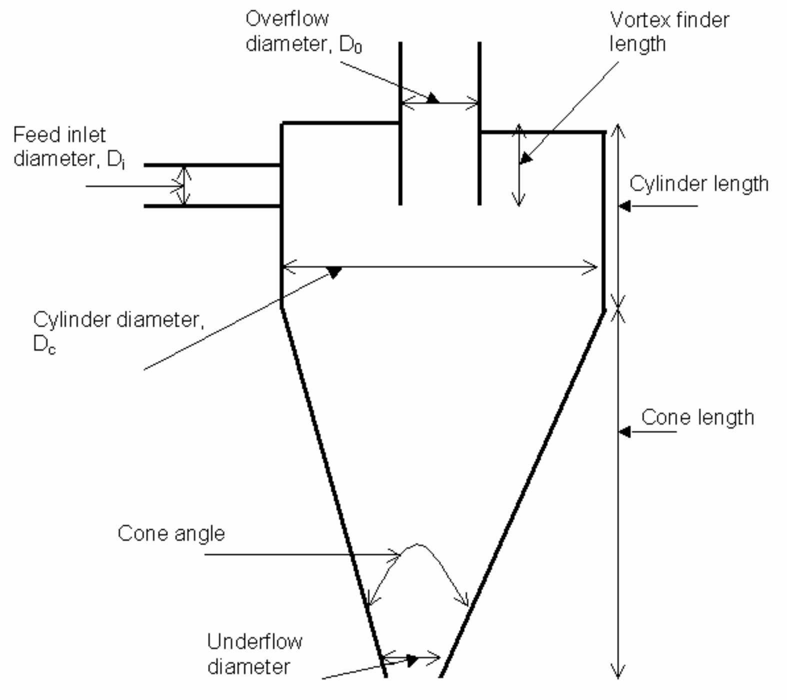

Figure 1.2 Hydrocyclone nomenclature.

The existence of an outer region of downward flow and an inner region of upward flow necessitates a position at which there is no vertical velocity. This applies throughout the greater part of the cyclone body, and an envelope of zero vertical velocity should exist throughout the body of the cyclone.. Particles thrown outside the envelope of zero vertical velocity by the greater centrifugal force exit via the underflow, while particles swept to the centre by the greater drag forces have an equal chance of reporting either to the underflow or overflow.

1.12 Basic Parameters for Standard Cyclone

The definition of a 'standard cyclone' is that cyclone which has the proper geometrical relationship between the cyclone diameter, inlet area, vortex finder, apex orifice, and sufficient length providing retention time to properly classify particles. The basic parameters of interest for a standard hydrocyclone are:

a. The inside diameter, Dc of the cylindrical feed chamber. This is the major parameter.
b. The area of the inlet nozzle at the point of entry into the feed chamber. This inlet nozzle is normally a rectangular orifice with the larger dimension parallel to the axis of the cyclone. The basic area of the inlet nozzle is approximately 0.05 times the cyclone diameter squared.
c. The vortex finder, which has the primary function of controlling both the separation and the flow leaving the cyclone. It should be sufficiently extended below the feed entrance to prevent short-circuiting of the material directly into the overflow. The size of the vortex finder equals 0.35 times the cyclone diameter.
d. The Cylindrical Section: This is located between the feed chamber and the conical section. It has the same diameter as the feed chamber and functions to lengthen the cyclone and increase the retention time for the basic cyclone, the length of the cylindrical portion is usually equal to 100% of D_C
e. Cone Section: The included angle of the cone section is normally between 20^0 and 10^0 and like the cylinder section, provides retention time. The termination of the cone section is the apex orifice. The critical dimension is the inside diameter at the discharge point. The size of the orifice is determined by the application involved and must be large enough to permit the solids that have been classified to underflow to exit the cyclone without plugging. The normal minimum orifice size would be 10% of the cyclone diameter but can be as large as 35%.

As can be seen from the above, the cyclone diameter D_c influences greatly both the separation efficiency and the flow rate - pressure drop relationships. D_c is a primary design variable to which all other dimensions are usually related.

With regard to the effect of the other dimensions of a hydrocyclone, a great deal has been written in the literature (Bradley, 1965, and Reitema, 1961) and the following is a brief summary of the most important design guidelines.

The feed inlet may be either rectangular or circular; the latter is simpler in construction but the former, a rectangular inlet with its long side parallel to the axis of the hydrocyclone, has been found to be most satisfactory (Bradley, 1965). The top of the feed entry should be flush with the roof of the cyclone in order to eliminate a dead space which would cause short-circuiting of the feed. The optimum diameter (or equivalent diameter by area) of the inlet is $D_c/4$ (Rietema,

1961) or $D_c/7$ (Bradley, 1965) for separation and $D_c/7$ (Rietema, 1961) for classification.

Long cyclones generally give higher capacities, Rietema(1961) found an optimum length of $L/D_c = 5$ for separation and $L/D_c = 2.5$ for classification. The included angle of the conical section is usually between 10 and 20 degrees. The optimum inside diameter of the vortex finder is $D_c/3$ for separation and $D_c/7$ for classification according to Rietema (1961) or $D_c/5$ according to Bradley (1965). An increase in the length of the vortex finder improves the efficiency of removal of the coarse particles but decreases the efficiency for the finer particles. The sharpest classification is obtained for vortex finder length in the range 0.33 - $0.4D_c$.

There is evidence favoring a smooth interior surface finish; abrasion resistance should be built into a hydrocyclone if it is to be operated with abrasive solids - a wide range of construction materials (steel, nylon, ceramics, polyurethane rubber etc) has been used, including the use of rubber liners.

Classification is aided by the use of collection pots or `grit boxes'; it is markedly improved by water injection (Kelsall *et al*,1975) in the conical section.

1.2 DESIGN PARAMETERS

1.21 Cyclone Efficiency

The commonest method of representing cyclone efficiency is by a “performance” or “partition” curve, shown in Figure 1.3. This curve relates the weight fraction or percentage, of each particle size in the feed which reports to the apex or underflow, to the particle size. The ‘cut point’ or separation size, of the cyclone is often defined as that point on the partition curve for which 50% of particles in the feed of that size report to the underflow, i.e. particles of this size have an equal chance of going either with the overflow or underflow, (Svarovsky, 1984). This point is usually referred to as the d_{50} size. The sharpness of the cut depends on the slope of the central section of the partition curve; the closer to vertical is the slope, the higher is the efficiency. The slope of the curve can be expressed by taking the points at which 75% and 25% of the feed particles report to the underflow. These are the d_{75} and d_{25} sizes, respectively. The efficiency of separation, or the so-called imperfection ‘I’, is then given by

$$I = \frac{d_{75} - d_{25}}{2d_{50}} \tag{1.1}$$

Many mathematical models of hydro-cyclones include the term “corrected d_{50}” taken from the “corrected” classification curve. It is assumed that in all classifiers, solids of all sizes are entrained in the coarse product liquid by short-circuiting in direct proportion to the fraction of feed water (liquid split) reporting to the underflow. Then each size fraction of the actual recovery curve is adjusted by an amount equal to the liquid recovery to reproduce the “corrected recovery curve (Figure 1.3) using the relation:

$$y' = \frac{y - R_f}{1 - R_f} \tag{1.2}$$

where y' is the corrected mass fraction of a particular size reporting to underflow, y is the actual mass fraction of a particular size reporting to the underflow, and R_f is the fraction of the feed liquid which is recovered in the coarse product stream.

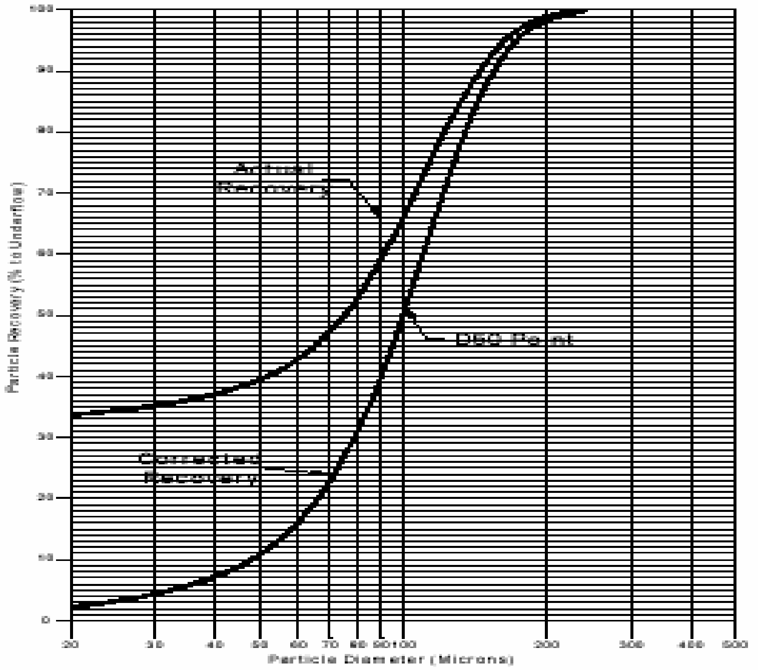

Figure 1.3 Particle recovery curve.

Investigations have shown that the **reduced recovery curve** remains constant over a wide range of cyclone diameters and operating conditions when applied to a slurry containing solids of a single specific gravity and a typical or normal size distribution as encountered in most grinding circuits.

Equation (1.2) gives a mathematical relationship which can be used to calculate the reduced recovery. This recovery, along with the by-passed solids, is used to predict the complete size distribution for the underflow product.

1.211 Prediction of Hydrocyclone Efficiency at Low Concentration

Various theories exist for the prediction of hydrocyclone efficiency operated at low input concentrations and low underflow-to-throughput ratios. Because of the rather complex flow profile and also some phenomena affecting the separation in hydrocyclones which are not yet fully understood (such as short circuiting of the feed), the approach by most authors has been semi-empirical, using measured velocity profiles and flow rate- pressure drop relationships.

The practical performance predictions, which normally assume Stokes' law for the calculation of the settling velocities, are commonly carried out in two steps. Firstly the cut size d_{50} is determined from existing theories and equations, and secondly the grade efficiency curve is deduced from available empirical formulae in the form of $G(d/d_{50})$.

1.212 Prediction of cut size d_{50}

According to the "residence time-theory" (Svarovsky, 1984) a particle ends up in the underflow if it has enough time to reach the walls of the cyclone. This time is called the "residence time" of the particle, equivalent to the time that elapses from particle entry into cyclone to its exit from the underflow. The driving force for this transfer is the normal acceleration due to the vortex (centrifugal force), and the velocity of transfer is the "sinking speed" due to this acceleration.

As the sinking speed depends on particle size, the probability of separation depends on particle size too. According to Svarovsky (1984), "*Cut size d_n is a diameter d for particles which have a probability of n% ending up in the underflow*". So the cut size d_{50} is a particle size which has a probability of 50% to end up in underflow.

For a hydrocyclone of given design proportions, operated at low input concentrations and at low underflow-to-throughput ratios *U/Q,* the cut size d_{50} may be assumed to depend only on the cyclone diameter D_c, the liquid viscosity, μ and density ρ, the difference between the density of the solids and of the liquid $\Delta\rho = \rho_s - \rho$, and suspension flow rate Q so that

$$d_{50} = f(D_c, \mu, \rho, \Delta\rho, Q) \tag{1.3}$$

It can be shown by dimensional analysis that the function in Equation (1.3) leads to a relationship between three dimensionless groups:

$$\left[\frac{d_{50}}{D_c}\right] = f\left(\frac{Q\rho}{D_c\mu}, \frac{\Delta\rho}{\rho}\right) \tag{1.4}$$

or alternatively

$$\left[\frac{d_{50}}{D_c}\right] = f\left(\mathrm{Re}_i, \frac{\Delta\rho}{\rho}\right) \tag{1.5}$$

where Re_i is the Reynolds number at the inlet, given by

$$\mathrm{Re}_i = \frac{v_i \rho D_i}{\mu} = \frac{4Q\rho}{\pi D_i \mu} \tag{1.6}$$

and v_i is the mean velocity in the inlet and D_i is the inlet diameter.

For a given design, D_i is proportional to D_c i.e.

$$D_i = k_i D_c \tag{1.7}$$

hence Re_i can also be written as

$$\mathrm{Re}_i = \left(\frac{4}{\pi k_i}\right)\left(\frac{Q\rho}{D_c\mu}\right) \tag{1.8}$$

The actual form of the function in Equation 1.4 has to be derived from the equations describing the separation process. Two general theories have been proposed in the literature: the 'equilibrium orbit’ theory and the 'residence time' theory.

The equilibrium orbit theory considers an orbit at which a given particle size is in equilibrium between the centrifugal force (due to the tangential velocity) and the drag force (due to the radial, inward velocity). Different particle sizes have different equilibrium orbit radii and it is assumed that all particles outside the locus of zero vertical velocity (LZVV) will be separated and all particles inside the LZVV will be carried away in the overflow. The cut size d_{5O} is that size which has an orbit which coincides with the locus of zero vertical velocity.

From this equilibrium orbit theory, Bradley(1965), by making assumptions as to the position of the locus of zero vertical velocity and the radial velocity profiles, derived a relationship as follows :

$$d_{50} = \frac{3(0.38)^n D_i^2}{\alpha}\left(\frac{\tan(\theta/2)\mu(1-R_f)}{D_c Q\Delta\rho}\right)^{0.5} \tag{1.9}$$

where R_f is the underflow-to-throughput ratio and n and α depend on the cyclone design and the fluid properties (α is also dependent on Q). For the cyclone configuration given by Bradley (1965) (θ=9°, D_i; $=D_c/7$, $D_o=D_c/5$), α= *0.45 and n* = 0.8.

If $R_f \rightarrow 0$ the above equation then becomes

$$\frac{d_{50}}{D_c} = 17.6x10^{-3}\left(\frac{\mu D_c}{Q\Delta\rho}\right)^{0.5} \tag{1.10}$$

This is of course a very simple form of the function in Equation 1.4; it can also be written as (from Equation 1.8)

$$\frac{d_{50}}{D_c} = 5.254x10^{-2}\left(\frac{1}{\mathrm{Re}_i(\Delta\rho/\rho)}\right)^{0.5} \tag{1.11}$$

which is in turn a specific form of Equation 1.5.

The other approach, the 'residence time' theory, considers the time taken by a particle to travel from the inlet to the cyclone wall. Rietema (1961) assumed that d_{50} is the size of particle which, if injected precisely in the centre of the inlet, just succeeds in reaching the cyclone wall at the apex. This is an approach similar to that used in the theory of gravity settling tanks and settling chambers. This method also allows calculation of other points on the grade efficiency curve because, as Rietema showed, for a circular inlet :

$G(1.41\ d_{50}) = 1.0$ $G(1.22\ d_{50}) = 0.8$
$G(0.71\ d_{50}) = 0.2$ and $G(0.56\ d_{50}) = 0.1$

This does not take into account the effect of turbulent diffusion and an appropriate correction has to be applied to the coarse part of the curve as shown by

Rietema(1961) (d_{50} is not appreciably affected by turbulent diffusion (Rietema (1961)).

Using this 'residence time' theory and experimental flow profiles measured by Kelsalll, Rietema derived a characteristic cyclone number Cy_{50}:

$$Cy_{50} = \frac{d_{50}^2 \Delta\rho}{\mu} L \frac{\Delta p}{\rho Q} \tag{1.12}$$

where L is the length of the cyclone.

Rietema proved experimentally that this number only depends on the geometric proportions of the cyclone. By varying the cyclone proportions in his experiments he optimized the design and found a minimum value of $Cy_{50} = 3.5$ for the following optimum set of cyclone relative dimensions:

$$\frac{L}{D_c} = 5, \quad \frac{L}{D_0} = 0.4, \quad \frac{D_i}{D_c} = 0.28, \quad \frac{D_0}{D_c} = 0.34$$

Clearly, if Equation 1.12 is to be used for cyclone design, it must be supplemented by a Δp - Q relationship (pressure drop – flow rate relationship); Rietema gave a graph of the pressure loss factor as

$$\frac{\Delta p}{\frac{1}{2}\rho v_i^2} = f(\mathrm{Re}_i) \tag{1.13}$$

for his optimum cyclone configuration. Gerrard and Liddle (1975) fitted a function to Rietema's data and found

$$\frac{\Delta p}{\frac{1}{2}\rho v_i^2} = 0.093\,\mathrm{Re}_i^{0.3748} \tag{1.14}$$

for $\mathrm{Re}_i > 5000$.

If this is substituted for Δp in Equation (1.12),

$$v_i = \frac{4Q}{\pi D_i^2} \tag{1.15}$$

an equation similar to Equation 1.9 is obtained :

$$\frac{d_{50}}{D_c} = 0.239\,\mathrm{Re}_i^{-0.1874}\left(\frac{D_c\mu}{Q\Delta\rho}\right) \tag{1.16}$$

As can be seen, the constant in Bradley's Equation (1.11) is now a function of Re_i; for $\mathrm{Re}_i = 10^5$ (a usual value in practice) this constant becomes 27.63×10^{-3}. Alternatively, Equation (1.16) can be expressed in a similar form to Equation (1.11):

$$\frac{d_{50}}{D_c} = 0.51\left(\frac{1}{\mathrm{Re}_i^{1.3748}(\Delta\rho/\rho)}\right)^{0.5} \tag{1.17}$$

1.221 Prediction of Corrected Cut Size, d'_{50}

Massarani (1997) has put forth the following correlation for the prediction of corrected
(or reduced) cut size (d'_{50}):

$$\frac{d'_{50}}{D_c} = K\left[\frac{\mu D_c}{Q(\rho_s-\rho)}\right]^{0.5} F(R_f)G(V) \tag{1.18}$$

where

$$F(R_f) = \frac{1}{1 + 1.73R_f} \tag{1.19}$$

$$G(V) = e^{4.5V} \tag{1.20}$$

and

$$R_f = B\left(\frac{D_u}{D_c}\right)^C \tag{1.21}$$

In the above equations (1.18 – 1.21),

R_f = the underflow- throughput- ratio
V = the volumetric feed concentration
D_u = underflow diameter
and B and C are constants for a given cyclone design.

It should be noted that the effect of the underflow-to-throughput ratio and the influence of the solids concentration on cut size were both considered in the equations. For Bradley's and Rietema's optimum design, Massarani (1997) obtained the parameters shown in Table 1.1.

Table 1.1. K, B and C constants for Bradley's and Rietema's conventional design.

Design	K	B	C	Eu

Bradley	0.016	54.6	2.61	7000
Rietema	0.039	145	4.75	1200

1.3 GRADE EFFICIENCY CURVES

The only complete description of the efficiency of a hydroclone is provided by a grade efficiency curve (G(d)). Hydrocylones are usually used with underflow-to-throughput ratios higher than 1-2% and it is therefore common to apply the reduced efficiency concept, whereby only the net (centrifugal) efficiency G'(d) is considered, with the contribution of the dead flux, which results from the volumetric split of the flow, removed. Yoshioka and Hotta(1955) obtained a curve to which Bradley (1965) later fitted an analytical function in the form of the Rosin - Rammler - Bennett equation:

$$G'\left(\frac{d}{d_{50}}\right) = 1 - \exp\left[-\left(\frac{d}{d_{50}} - 0.115\right)^3\right] \tag{1.22}$$

for $0.002 < G' < 0.98$.

Similar observations to those of Yoshioka and Hotta were made by Lynch et al (1974) who deduced another equation.

$$G'\left(\frac{d}{d_{50}}\right) = \frac{\exp(\alpha d / d_{50}) - 1}{\exp(\alpha d / d_{50}) + \exp(\alpha) - 2} \tag{1.23}$$

and found $\alpha = 4.9$ for all their tests on geometrically similar hydrocyclones ranging from 10.2 to 38.1 cm in diameter, with feed concentrations of limestone of 22 to 31% v/v. The constant α was found to depend on the material used, Lynch's previously published work (1968) give $\alpha = 2.5$ for silica ore and $\alpha = 2.0$ for copper ore. Equation (1.23) is also referred to as exponential sum

On the basis of the available experimental evidence it is reasonable to expect that the shape of the reduce grade efficiency curve $G'(d/d_{50})$ may be reasonably constant (for a given cyclone design (at low particle concentrations), but its slope at higher volume concentrations becomes dependent on the feed material. This is also to be expected from the analogy with hindered settling in gravity field. Equation (1.22) or (1.23) gives useful guide for estimating purposes; a knowledge of d_{50} is of course necessary for a full curve of G'(d) to be obtained.

1.31 Pressure Drop

The pressure drop in a hydrocyclone is an important operational variable; normally only the static pressure loss is taken into account (this consists of the friction losses and the centrifugal head), thus assuming that the kinetic energy of the feed is the same as that of the overflow and that the latter is recoverable.

As with the efficiency there are numerous pressure drop correlations in the literature. One such equation that can be found frequently in the literature (Dahlstrom (1954)) is

$$Q = 278(D_i x D_0)^m \sqrt{\frac{\Delta p}{\rho_i}} \;\; (cm^3 s^{-1}) \qquad (1.24)$$

where D_o and D_i are in cm, Δp in *atm* and ρ_i is the specific gravity of the suspension compared with water. Many authors take **m = 1**.

The correlation in Equation (1.24) reduces to

$$Q = 278k\frac{D_c^2}{12}\sqrt{\frac{\Delta p}{\rho_i}} = 23.2kD_c^2\sqrt{\frac{\Delta p}{\rho_i}} \qquad (1.25)$$

where the coefficient **k** is related to friction factor λ, the effective cyclone diameter D, the length L, and a geometrical factor ξ (Battaglia, 1962) by the following equation:

$$k = \sqrt{\frac{1}{\lambda}\frac{D}{L}\frac{1}{\xi}}$$

and

$$\xi = \frac{1}{3}\left(\frac{D_i}{D_0} + 1 + \frac{D_0}{D_i}\right)$$

if Rietema's optimum design proportions are used. The geometrical factor ξ is then equal to 1.013. Although Equation 1.25 (and many other correlations available in the literature) seemingly gives $\Delta p \sim Q^2$, which would indicate a constant pressure loss coefficient ($2\Delta p / v^2{}_i p_i$); in practice this coefficient depends on Reynolds number Re_i which leads to the constant k in Equation (1.25) being a function of Re_i and hence of the flow rate, Q. This dependence exists in Equation (1.25) because the friction factor λ is a function of Re_i.

Bradley proposed a relationship which describes the pressure drop behaviour of the chosen design of a cyclone as

$$\Delta p = k_p \frac{Q^{2.35}}{D_c^{4.7}} \tag{1.26}$$

where k_p depends on the cyclone configuration and units used for Q and D_c. Svarovsky (1977) indicated that

$$\Delta p = k_s \frac{Q^{3.26}}{D_c^{4.7}} \tag{1.27}$$

if the same exponent of *4.7* is adopted. Comparison of Equation 1.26 and 1.27 suggests that the exponent on Q also depend on the cyclone configuration. The constant k_s is a function of the concentration **c** (v/v) as follows:

$$k, = (4.76 - 0.451 \text{ Ln } \boldsymbol{c})\ 10^{8} \ (\text{kg m}^{-6.08}\text{s}^{1.26}) \tag{1.28}$$

for $0.003 < c < 0.15$ v/v (i.e., $0.3\% < c < 15\%$ v/v)

1.4 DESIGN AND SCALE-UP OF HYDROCYCLONES AT LOW FEED CONCENTRATION

The literature on hydrocyclones is full of studies of the effects of the relative geometrical proportions, such as the inlet and outlet diameters, the cyclone length, the vortex finder length and the cone angle, on pressure drop or capacity and separation efficiency. Using this information, an engineer faced with the design of a hydrocyclone installation could select a geometry which in his view would give him the best performance. The problem is that his cyclone might be unique, with a combination of relative proportions never used or tested by anybody before, so that he could not reliably predict its performance. There are, of course, many theories and models available but these only give approximate estimates unless a specific cyclone is adopted.

By far the best method is to select a `standard' or known design of cyclone defined by a set of relative geometrical proportions and only then can a meaningful scale-up be carried out which leads to reasonably reliable designs. There is a choice of several standard or optimum designs developed and tested by different researchers as well as some well-documented commercial cyclones.

The following scale-up procedure is based on the available theories and relationships in dimensionless form with empirical constants obtained from test data. It should enable an engineer to make a qualified judgment as to whether a hydrocyclone would be a viable proposition for his process and, if so, to design (and make) his own unit. Even in cases when he prefers to go to a specialized manufacturer and buy a commercially available hydrocyclone assembly, the data given in the commercial literature allow the engineer to do his homework which then puts him in a much stronger position in dealing with the manufacturer.

1.41 Design based on required flowrate Q and cut size d_{50}, at a given pressure drop ΔP

The first decision to be made is the type of standard cyclone to be used. Some guidance in this and many known designs are summarized in Table 1.2. The problem is then reduced to the selection of the diameter and the number of cyclones to be used in parallel to meet the requirements in terms of total flow rate and separation efficiency (or cut size). The operating pressure drop has to be given here otherwise there is a number of possible solutions.

The separation efficiency is described here by the cut size d_{50} and it is assumed that d_{50} is known either from the requirement of the minimum mass recovery or from process requirements in classification applications. Note that if the cut size is not known it can be found from an optimization procedure between the running costs and the cost of lost product.

1.411 Scale-up at low feed solids concentrations

At low solids concentrations (less than 1% by volume) the flow pattern in the cyclone is not affected by the presence of particles in the flow and particle-particle interaction is negligible. Consequently, as only few particles report to the underflow, the underflow-to-throughput ratio can be kept low and it can also be assumed to have no effect on the cut size d_{50}.

Dimensional analysis coupled with two theories of separation in hydrocyclones (equilibrium orbit theory and residence time theory - Equations (1.11) and (1.16), respectively) gives two basic relationships between three dimensionless groups:

$$Stk_{50}.Eu = Const \tag{1.29}$$

$$Eu = K_p \mathrm{Re}^{n_p} \tag{1.30}$$

(K_p and n_p are empirical constants), where the Reynolds number (based on D_c) is defined as

$$\mathrm{Re} = \frac{v\rho D_c}{\mu} \tag{1.31}$$

The Euler number is the well-known pressure loss factor defined as

$$Eu = \frac{\Delta p}{\rho v^2 / 2} \tag{1.32}$$

Stk_{50} is the Stokes number defined as

$$Stk_{50} = \frac{d_{50}^2 \Delta\rho v}{18\mu D_c} \tag{1.33}$$

where $\Delta\rho$ is the difference in density between solute and solvent, and v is the characteristic velocity calculated from cross-section of the cyclone body, i.e.

$$v = \frac{4Q}{\pi D_c^2} \tag{1.34}$$

and the other variables are cyclone diameter D_c, liquid viscosity μ, and density ρ, density difference between the solids and the liquid $\Delta\rho$ and suspension flow rate Q.

Note: It should be pointed out here that some authors base their analysis on different reference velocities and characteristic dimensions. Inlet velocity and inlet diameter are frequently used, leading to `inlet' Reynolds number Re_i, Stokes number Stk_i and Euler number Eu_i, and some authors define other Reynolds numbers. For geometrically similar hydrocyclones in a scale-up procedure the choice of characteristic velocity and dimension is arbitrary, as long as it is specified. The basis adopted here directly involves the most important cyclone dimension - its diameter D_c - and it therefore gives the simplest and most direct route to scale-up calculations.

Table 1.2 gives a review of the well-known cyclone designs in terms of the geometrical proportions and constants for Equations 1.29 and 1.30, the latter being determined from test results. The use of Equations 1.29 to 1.34 in cyclone design is best illustrated in numerical examples, as follows:

Table 1.2: Summary of some known Hydrocyclone Designs

Cyclone type and size of hydrocyclone	D_iD_c	D_oD_c	l/D_c	L/D_c	Angle θ, degrees	$Stk_{50}Eu$	K_p	n_p	$Stk^{4/3}{}_{50}Eu$
Rietema's design (optimum separation), $D_c = 0.075$ m	0.28	0.34	0.4	5	20	0.0611	24.38	0.3748	$2.6\text{x}10^{-3}$
Bradley's design, $D_c = 0.038$m	0.133 (1/7.5)	0.20 (1/5)	0.33 (1/3)	6.85	9	0.1111	446.5	0.323	$2.76\text{x}10^{-3}$
Mozley cyclone, D_c - 0.022m	0.154 (1/6.5)	0.214 (3/14)	0.57 (4/7)	7.43	6	0.1203	6381	0	$3.20\text{x}10^{-3}$
Mozley cyclone, $D_c = 0.044$m	0.160 (1/6.25)	0.25 (1/4)	0.57 (4/7)	7.71	6	0.1508	4451	0	$4.88\text{x}10^{-3}$
Mozley cyclone, $D_c = 0.044$m	0.197 (1/5)	0.32 (1/3)	0.57 4/7)	7.71	6	0.2182	3441	0	$8.70\text{x}10^{-3}$
Warman 3" Model R, D_c = 0.076m	0.29 (1/3.5)	0.20 (1/5)	0.31	4.0	15	0.1079	2.618	0.8	$2.66\text{x}10^{-3}$
RW 2515 (AKW), $D_c = 0.125$	0.20 (1/5)	0.32 (1/3)	0.8	6.24	15	0.1642	2458	0	$7.14\text{x}10^{-3}$

Note: D_i = Feed inlet diameter, D_0 = Vortex diameter, D_c = cyclone diameter, l = vortex finder length, L = cyclone length. Source: (Svarovsky, L. Solid – Liquid Separation, 2ed. Butterworths, London (1981).

Example 1.1

A hydrocyclone is to be operated at a flow rate of 0.005m^3 s^{-1} with solids of density 300kgm^{-3} suspended in water of density 1000kgm^{-3} and viscosity of 0.001 Ns m^{-2} at concentrations of less than 1% by volume. The operating pressure available is 10^5 N m^{-2} and both underflow and overflow discharge into the atmospheric pressure. Find the cyclone diameter and estimate the operating cut size.

Solution

Combination of Equations 1.30, 1.31, 1.32 and 1.34 gives cyclone diameter as follows:

Substitute Eq.(1.31) into Eq.(1.30). Then substitute Eq.(1.32) to eliminate *Eu*. Now eliminate *v* by using Eq. (1.34). This will yield the following equation, after rearrangement:

$$D_c^{(4+n_p)} = \left(\frac{4Q}{\pi}\right)^{2+n_p} \left(\frac{\rho}{\mu}\right)^{n_p} \frac{K_p \rho}{2\Delta p} \quad (1.35)$$

Substituting values,

$$D_c = \left[\left[\frac{28 * 0.005}{22}\right]^{2.3748}\left(\frac{10^3}{0.001}\right)^{0.3748}\left[\frac{10^3(24.38)}{2x10^5}\right]\right]^{\frac{1}{4.3748}} = 0.1297m$$

The cut size is obtained from Equations 1.29 - 1.34 as follows:
Substitute Eq.(1.33) into Eq.(1.29). Then eliminate *Eu* using Eq.(1.32). Lastly, use Eq.(1.34) to eliminate v, rearrange and obtain the following equation:

$$d_{50}^2 = Stk_{50}.Eu\frac{36\mu Q\rho}{\pi\Delta p\Delta\rho D_c} \tag{1.36}$$

Using constants from Table 1.2, Equations 1.35 and 1.36 give for Rietema's cyclone (optimum separation) $D_c = 0.1297$ m and $d_{50} = 11.62\mu$m, for Bradley's cyclone D = 0.2233 m and $d_{50} = 11.93\mu$m. As can be seen from these results, Rietma's cyclone is smaller for the same cut size and is therefore likely to be cheaper in capital cost (and will occupy less space) than the Bradley's cyclone designed for the same operating conditions.

If smaller cut size is required, the total flow could be divided into a number of smaller cyclones operated in parallel. For higher number of cyclones, it is always found that Rietema's cyclones are always smaller than Bradley's for the same cut size and pressure drop.

If a condition of a maximum cut size is introduced in the example, the number of cyclones could be determined by trial and error from Equations 1.35 and 1.36. Thus, for example, if the cut size were to be no more than 8 µm, the minimum number of cyclones would be five.

Example 1.2

Determine the number and diameter of hydrocyclones (Rietema's optimum separation design) for the following operating conditions:
$\Delta p = 305.24$ kPa, $d_{50} = 8\mu$m, $\rho_s = 2640$kg m^{-3}, $\rho = 1000$kg m^{-3}, $\mu = 0.001$ Ns m^{-2}, Q = 8.333 x 10^{-3} m^{-3} s^{-1}, feed solids concentration less than 1% by volume.

Results

Two cyclones, $D_c = 91$mm.

1.5 DESIGN OF HYDROCYCLONES AT HIGH FEED CONCENTRATION

The feed concentration of solids is a very important variable, which is known to affect both the separation efficiency and the pressure drop of a given hydrocyclone. It is, however, a relatively neglected effect in terms of literature information and little is known about it quantitatively.

All we know is that, with increasing concentration, the separation efficiency goes down appreciably, i.e. the cut size increases. At higher input concentrations the underflow-to-throughput ratio R_f has to be increased (to allow greater volume of separated solids to be discharged) by opening up the underflow orifice; the factor R_f also affects the separation efficiency. The control of the underflow orifice in fact plays an important part in the operation of hydrocyclones and for that purpose most cyclones are equipped with either a continuously variable underflow orifice (operated mechanically or pneumatically) or a series of replaceable nozzles. The dilemma is that a thick underflow and a high solids recovery cannot be achieved at the same time and one has to be sacrificed for the other. For maximum solids recoveries, the solids concentration in the underflow should be below a certain limit because above that limit some solids will start going into the overflow. It means that this is at the expense of total separation efficiency. The practical limit of underflow concentrations achievable is about 50% by volume for materials like limestone or coal slurries. Some devices exist for automatic control of the underflow orifice in order to keep the underflow concentration below, or at, a certain value.

1.51 Semi-empirical Scale-up at High feed solids concentrations

The effects of changing operating and design parameters in cyclones are very complex in that all parameters are interrelated. It is almost impossible to select a cyclone to give the precise separation required and it is nearly always necessary to adjust feed inlet, vortex finder, apex opening and pulp pressure, and dilution. Designers, therefore, tend to specify cyclones capable of handling the flow rates required, with provision for fitting suitable ranges of feed, overflow, and underflow openings. There are a number of empirical relationships, which are used by designers in predicting performance and designing cyclones.

One method of design at high feed concentration is based on the use of dimensionless groups and Equations (1.29) to (1.34) apply. However, at higher concentrations, the feed concentration as a function of the volume, V (that is, the volumetric concentration), has to be included as an additional dimensionless group. One of the oldest is that of Dahstrom (1954);

$$d_{50} = \frac{13.7(D_o D_i)^{0.68}}{Q^{0.53}(\rho_s - \rho_L)^{0.5}} \tag{1.37}$$

where d_{50} is the cut-point (µm), D_o is the overflow diameter (cm), D_i is the inlet diameter (cm), Q is the total flow rate (m^3 h^{-1}), ρ_S is the specific gravity of solids, and ρ_L is the specific gravity of liquid.

Such equations are not, however, directly applicable to industrial scale cyclones, as most of the work was carried out on dilute slurries using very small diameter cyclones

Plitt (1976) developed a mathematical model which gives reasonable predictions of performance of large diameter cyclones, operating at high solids content, over a wide range of operating conditions. The model has been successfully applied to the development of automatic control systems in the comminution circuits of various Australian mines. Four fundamental parameters were determined in terms of the operating and design variables, these being the cut-size, the flow split between underflow and overflow, the sharpness of separation, and the capacity in terms of the pressure drop across the cyclone. The model formulated enables the complete cyclone performance to be calculated with reasonable accuracy without requiring experimental data.

The equation for the cut-size is:

$$d_{50(c)} = \frac{\left[14.8 D_c^{0.46} D_i^{0.6} D_0^{1.21} \exp(0.063V)\right]}{\left[D_u^{0.71} h^{0.38} Q^{0.45} (\rho_S - \rho_L)^{0.5}\right]} \tag{1.38}$$

where $d_{50(c)}$ is "corrected" d_{50} (µm): D_c, D_i, D_o, D_u are inside diameters of hydrocyclone, inlet, vortex finder and apex, respectively (cm); V is the volumetric percentage of solids in feed; h is the distance from the bottom of the vortex finder to the top of the underflow orifice (cm); Q is the flow rate of the feed slurry (m^3h^{-1}); and ρ_S, ρ_L are the density of solids, density of liquid, respectively(g/cm^{-3})

The equation for the volumetric flowrate of slurry to the cyclone is

$$Q = \frac{\left[0.021 \Delta P^{0.56} D_c^{0.21} D_i^{0.53} (D_u^2 - D_o^2)^{0.49}\right]}{\exp(0.031V)} \tag{1.39}$$

where ΔP is the pressure drop across the cyclone in kPa (1 psi = 6.895 kPa).

For preliminary design purposes, Mular and Jull (1978) have developed expressions from results obtained on Krebs cyclones (Anon, 1977), relating d_{50} to the operating variables for "typical" cyclones, of varying inside diameter. A "typical" cyclone has an inlet area of about 7% of the cross-sectional area of the feed chamber, a vortex finder of diameter 35-40% of the cyclone diameter, and an apex diameter normally not less than 25% of the vortex-finder diameter. The equation for the cyclone cut-point is:

$$d_{50(c)} = \frac{\left[0.77 D_c^{1.875} \exp(-0.301 + 0.0945V - 0.00356V^2 + 0.0000684V^3)\right]}{\left[Q^{0.6}(\rho_S - 1)^{0.5}\right]} \tag{1.40}$$

The maximum volume of slurry that the cyclone can handle is given by:

$$Q = 9.4 \times 10^{-3} \sqrt{\Delta P} D_c^2 \tag{1.41}$$

Equations such as these have been used in computer controlled grinding circuits to infer cut-points from measured data, but their use in this respect is declining with the increased application of on-line particle size monitors. Their great value, however, is in the design and optimization of circuits employing cyclones by the use of computer stimulation. For instance, Krebs Engineers applied mathematical models to a grinding-classification circuit and predicted that two-stage cyclone, as opposed to the more common single stage, would allow a 6 percent increase in grinding circuit through-put. Without the aid of cyclone modeling, such work would be costly and time-consuming (Apling *et al*, 1980; Edmiston, 1982). The equations can also be very valuable in the selection of cyclones for a particular duty, the final control of cut-point and capacity made by adjusting the size of inlet, vortex-finder, and apex.

Svarovsky and Marasinghe (1980) reported the following expression for the effect of high solids concentration in the feed:

$$StK_{50}(r) = k_1 (1 - R_f)\left(\exp(k_2 V)\right) \tag{1.42}$$

where StK_{50}(r) includes the previously described reduced cut size which takes the ‘dead flux’ effect into account whereby very fine particles simply follow the flow and split in the same ratio as the liquid. The correlation has proven to hold well above 8 volume percent, and the values of the constants k_1 and k_2 were found to be (9.05 x 10-5) and 6.461, respectively, for limestone and an AKW (Amber Kaolinwerke Gmbh, Hirschau, Germany) hydrocyclone of 125mm in diameter.

An exhaustive study of concentrations up to 10-vol. % was carried out by Medronho (1984) and he ontained the following equations:

$$StK_{50}(r)Eu = 0.047[in\,(\,{}^{1}\!/\!_{R_f}\,)]^{0.74}\exp(8.96V) \tag{1.43}$$

$$Eu = 71(\mathrm{Re})^{-0.116}\,({}^{D_i}\!/\!_{D_c})^{-1.3}\exp(2.12V) \tag{1.44}$$

$$R_f\ = 1218({}^{D_u}\!/\!_{D_c})^{-4.75}\,(Eu)^{-0.30} \tag{1.45}$$

where D_i, D_c , and D_u are the inlet, body and underflow diameters of the hydrocyclone, respectively.

For concentrations higher than 10-vol.%, many slurries show non-Newtonian behaviour and the Reynolds and Stokes numbers can be rewritten to consider such behaviour (Ortega-Rivas and Svarovsky, 2003). The correlations derived are the following (Ortega-Rivas, 2004):

$$StK^{*}_{50}\,(r)Eu = 0.006[in\,(\,{}^{1}\!/\!_{R_f}\,)]^{2.37}\exp(6.84V) \tag{1.46}$$

$$Eu = 1686(\mathrm{Re})^{-0.035}\,\exp(-3.39V) \tag{1.47}$$

$$R_f\ = 32.8({}^{D_u}\!/\!_{D_c})^{1.53}\,(\mathrm{Re}^{*})^{-0.34}\exp(3.70V) \tag{1.48}$$

where $StK^{*}_{50}(\ r)$ and Re* are the *generalized* Stokes and Reynolds numbers, meaning that they include the parameters of characterization of non-Newtonian suspensions, that is, the fluid consistency index, K' and the flow behavior index, n, instead of the medium viscosity (Ortega-Rivas and Svarovsky, 2003). The term *generalized* is used to imply that, for Newtonian suspensions, the Stokes and Reynolds numbers above would reduce to the common forms found in the literature.

1.52 Empirical Design

The design of a hydrocyclone to determine the number of units needed is based on the system flow rate. Where a single unit is not available or feasible, several small-capacity cyclones may be setup to run in parallel to meet the required flow rate.

Usually a cyclone is chosen based on recovery of a particle at a specific size and performance definitions. The point usually used for this is the d_{50} point, and is the basis of sizing factors. From the d_{50} point, the recovery of all particle sizes can be determined, and a recovery curve can be constructed.

The determination of the d_{50} point and subsequent recovery curve is based first on a base diameter (d_{50b}), which is the cyclone diameter at a set of standard conditions. The d_{50b} is the particle with a 50% chance of capture based only on cyclone diameter. The actual d_{50} corrects the d_{50b} for cyclone geometry and fluid properties.

1.521 Base Condition for Standard Hydrocyclone Performance

In determining the "proper size" and the number of cyclones required for a given application., it is necessary to establish a base-line condition as follows:

- Feed Liquid – water at 20°C
- Feed Solids – spherical particles of sp gr = 2.65
- Feed Concentration – less than 1% solids by volume
- Pressure drop – 69kPa (10 psi)
- Cyclone geometry – 'standard cyclone'

1.522 Design Equations

The relationship of the cyclone diameter to the d_{50b} can be determined by the following equation

$$d_{50_b} = 5.27D^{0.66} \tag{1.49}$$

where D is the cyclone diameter in inches.

In SI units, Equation 1.49 becomes

$$d_{50_b} = 2.84D^{0.66} \;,\quad D \textit{ is in cm.} \tag{1.50}$$

This equation shows that as the cyclone diameter increases, the base d_{50b} also increases. This relationship is shown in Figure 1.4

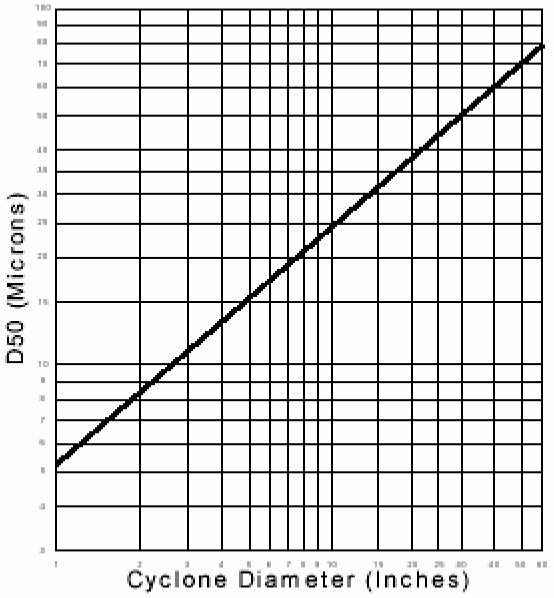

Figure 1.4 Cyclone diameter vs. D_{50} (for 'typical' cyclone).

1.523 Correction Factors

These are needed because the standard d_{50b} is based on feed liquid water at base conditions. This base d_{50b} is adjusted by the use of correction factors for the process variables. Therefore the actual d_{50} becomes

$$d_{50} = d_{50_b} C_1 C_2 C_3 ..etc \tag{1.51}$$

where

C_1 = Correction factor for feed solids concentration
C_2 = Correction factor for solids specific gravity
C_3 = Correction factor for pressure drop across cyclone

These are the three major correction factors that affect the performance and sizing of hydrocyclones.

1.524 Solids Concentration

If the concentration of the feed solids increases, both the slurry viscosity and the specific gravity increase proportionally. This tends to inhibit separation and causes the d_{50} to increase. The relationship can be determined by the following equation:

$$C_1 = \left(\frac{(53-V)}{53} \right)^{-1.43} \tag{1.52}$$

where V is the percent solid by volume of the cyclone feed. This is shown in Figure 1.5.

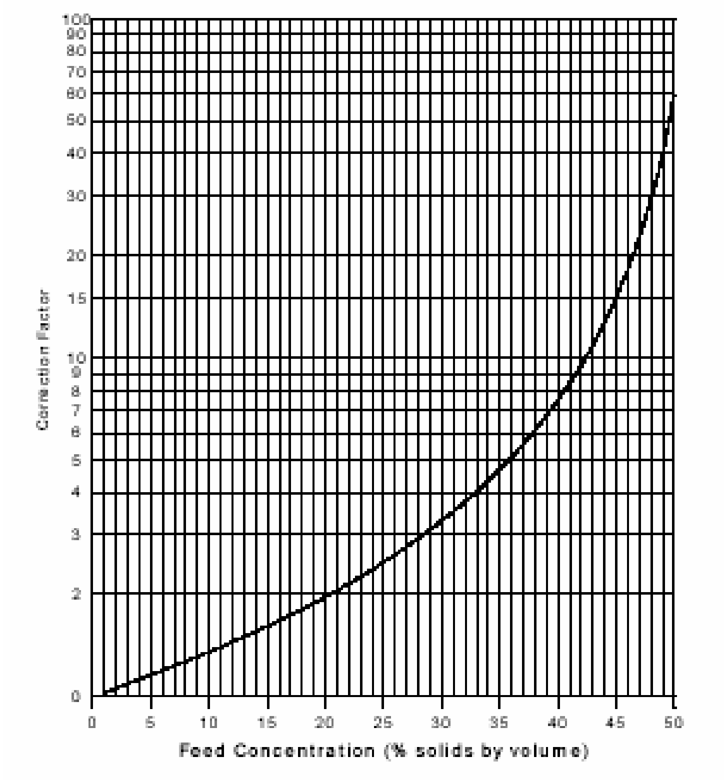

Figure 1.5 Correction for Feed Concentration.

The figure indicates that the level of percent solids is extremely important in determining the proper separation, as the higher the concentration the coarser the separation. It should be pointed out that this correction is a relative measure of slurry viscosity and is affected by such things as the size of particles present as well as particle shape. For example, a feed that contains large amount of clay would tend to shift this curve to the left and result in a coarser separation, whereas the absence of fines would shift the curve to the right and result in a finer separation. Many other variables such as liquid viscosity also affect this correction.

1.525 Specific gravity

The second correction factor is for specific gravity. This correction factor is based on the difference in specific gravity between the solid particles and the liquid phase. As the difference increases, the separating force also increases. Since the cyclone does not actually achieve a size separation but rather a mass separation, the specific gravity of the particle is extremely important in determining the separation. It has been found that Stoke's law can be applied to determine particle diameters which would produce the same terminal setting velocity for a particle of known specific gravity in a liquid of known specific gravity as compared to a particle of 2.65 specific gravity in water. The correction is determined by the following equation:

$$C_2 = \left(\frac{1.65}{(\rho_S - \rho_L)}\right)^{-1.43} \tag{1.53}$$

where

ρ_S = Solids specific gravity
ρ_L = Liquid specific gravity

Figure 1.6 is a graph of this effect.

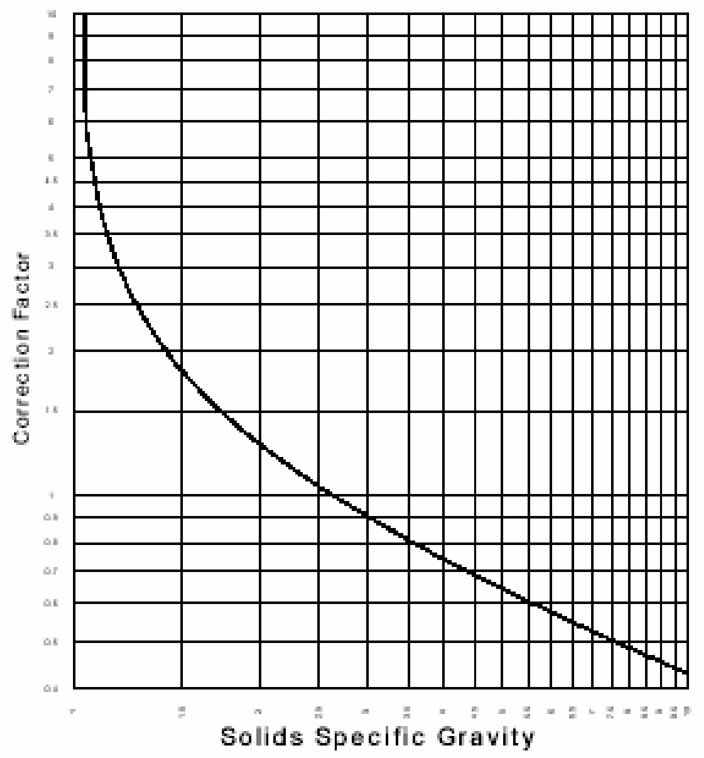

Figure 1.6 Correction for solids specific gravity (in water).

1.526 Pressure Drop

The last correction factor is for pressure drop across the cyclone. Pressure drop is a measure of the energy being utilized in the cyclone to achieve the separation. It is recommended that pressure drops, wherever possible, be designed in the 40 to

70 kPa (5 to 10 psi) range to minimize energy requirements as well as reduce wear rates.

As the pressure drop increases, the centripetal forces also increase, allowing for a finer separation. This relationship is described by

$$C_3 = 1.91\Delta P^{-0.28} \tag{1.54}$$

where ΔP is the pressure drop in psi (pounds per square inch).

Again in SI units, this correction factor becomes

$$C_3 = 3.27\Delta P^{-0.28},\ \Delta P\ \ is\ \ in\ \ kPa \tag{1.55}$$

Figure 1.7 shows this relationship. As indicated, a higher pressure drop would result in a finer separation and lower pressure drop in a coarser separation.

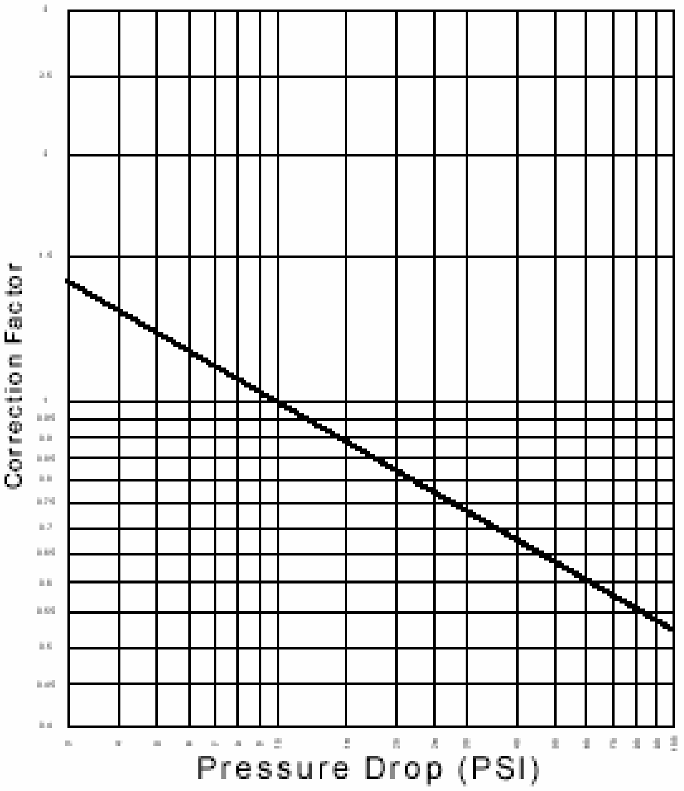

Figure 1.7 Correction for solids specific gravity (in water).

1.527 Apex selection

The proper selection of apex size is critical to proper hydrocyclone performance. For each application, a circulating load is normally given which establishes the amount of solids which must pass through the hydrocyclone underflow. Experience shows that an underflow density of 50% to 53% solids by volume is typical for primary grinding circuits, whereas an underflow density of 40% to 45% by volume is normal for regrind circuits. Therefore, an underflow density can be assumed which establishes the total flow that must report through each hydrocyclone apex. Figure 1.8 shows the approximate flow rate for a given diameter apex orifice.

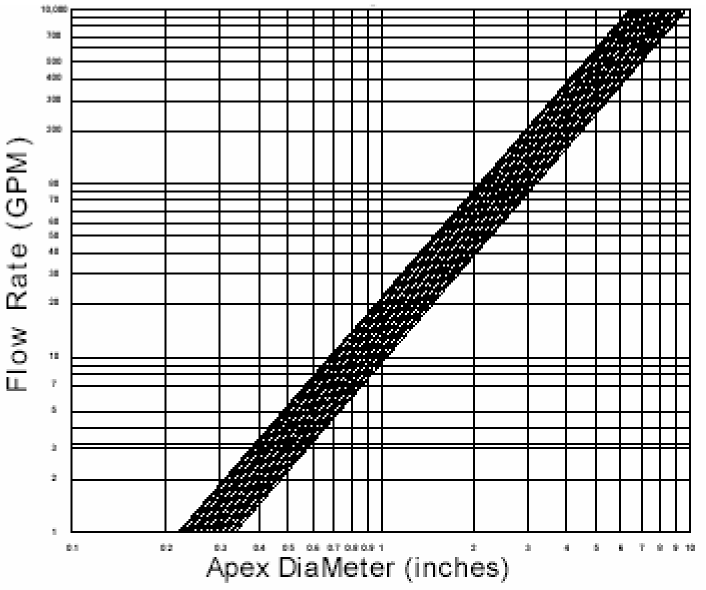

Figure 1.8 Apex diameter versus Flow rate.

1.53 Classification

Historically, classification has been defined as the particle size of which 1% to 3% report to the cyclone overflow with coarser particles reporting to the cyclone underflow. Recent investigations have defined classification as the particle size of which 50% reports to the underflow and 50% to the overflow, or the d_{50} point. Fig 1.3 shows the typical relationship between the particle diameter and the percent recovered to the underflow.

The portion of the curve near the 50% recovery level is quite steep and lends itself readily to determining an accurate particle diameter. The figure also shows that the actual recovery curve does not decrease below a certain level. This indicates that a certain amount of material is always recovered to the underflow and bypasses classification. If a comparison is made between the minimum recovery level of solids to the liquid that is recovered, they are found to be equal. Therefore, it is assumed that a percent of all size fractions reports directly to the underflow as bypassed solids in equal proportion to the liquid split. Then each size fraction of the actual recovery curve is adjusted by an amount equal to the liquid recovery to produce the 'corrected recovery' shown in Figure 1.3.

As the d_{50} point changes from one application to another, the recovery curves shift along the horizontal axis. In order to determine a single graph which represents the corrected recovery curve, the particle of each size fraction is divided by the d_{50} value and a 'reduced recovery' curve can be plotted as shown in Figure 1.9. This figure is based on the Lynch-Rao equation which is shown below.

$$R(d/d_{50}) = \frac{100\left[e^{\left(4d/d50\right)} - 1\right]}{\left(e^{\left(4d/d50\right)} + e^{4} - 2\right)} \tag{1.56}$$

where $R(d/d_{50})$ = recovery at a specific particle size. This equation does not account for the misclassification of material that occurs due to the hydraulic nature of the cyclone, which causes some of the finer, lighter particles to report to the underflow based on the liquid flow split.

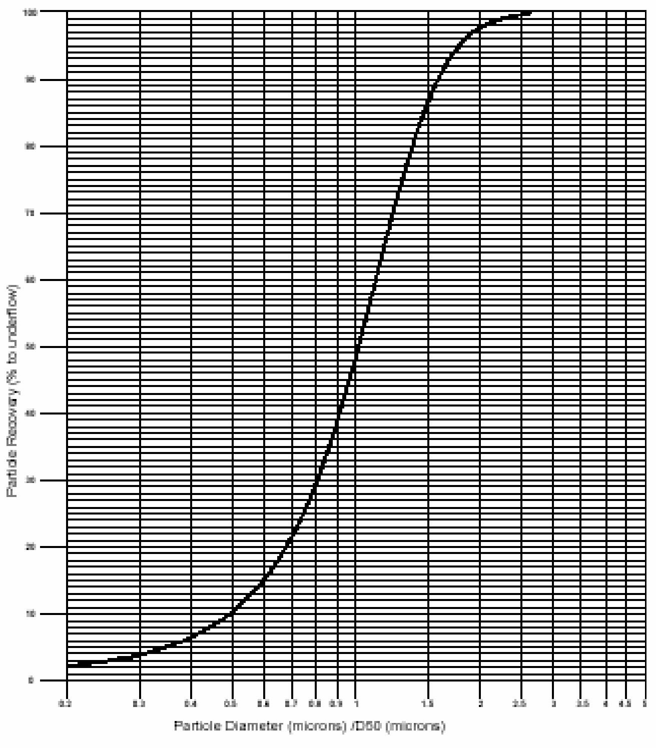

Figure 1.9 Reduced particle recovery curve.

However for initial sizing considerations, the bypass of fines can be neglected and the Lynch-Rao equation, shown above, should be adequate.

In addition to the above variables, the cyclone geometry and internal fittings can be changed to effect the classification. Separation is affected by three variables in the cyclone geometry – the inlet head design, vortex finder size and overall cyclone length.

1.54 Design Procedures

The design procedures depend on the variables that are specified, and there are generally two cases to consider. However, in both cases, the first decision to be made is the type of standard cyclone to be used (see Table 1.2). Then there are two cases to consider as shown below.

1.541 Design Based on Flow Rate and Cut Size at given Pressure Drop

After choosing the standard cyclone to be used, the problem then reduces to the selection of the diameter and number of cyclones to meet process requirements in terms of total flow rate and separation efficiency (or cut size). The operating pressure drop is required otherwise there exists a number of solutions. The flowchart is shown in Figure 1.10.

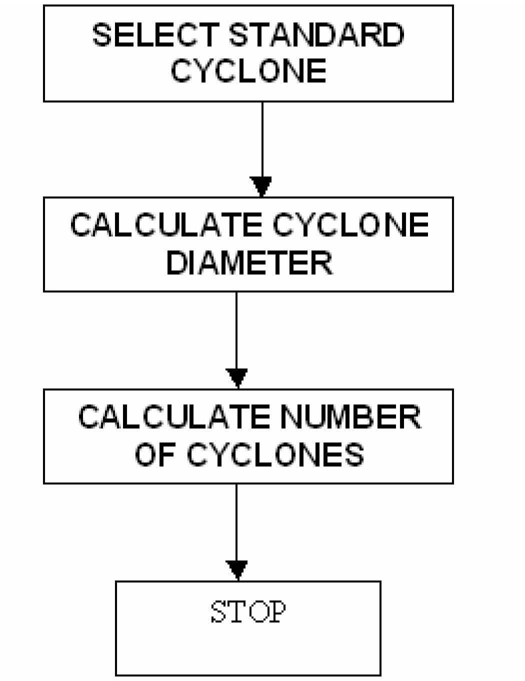

Figure 1.10 Design Procedure based on Q, d_{50} and ΔP.

1.542 Calculational Procedure

The design procedure can be implemented as follows

1. Choose a standard cyclone
2. Calculate C_1, C_2, C_3
3. Calculate cyclone diameter
4. Obtain the flow rate given the diameter and the pressure drop
5. Obtain the number of cyclones required for the duty.

The relationship between pressure and flow rate is shown in Figure 1.11. The flow rate (item 4 above) can be calculated using the following, appropriate equations which were regressed from Figure 1.11:

$$Q = 11\Delta P^{0.5488} \quad \textit{for}\ 4''\textit{-diameter cyclone} \tag{1.57a}$$

$$Q = 23\Delta P^{0.5861} \quad \textit{for}\ 6''\textit{-diameter cyclone} \tag{1.57b}$$

$$Q = 44\Delta P^{0.61498} \quad \textit{for}\ 10''\textit{-diameter cyclone} \tag{1.57c}$$

$$Q = 110\Delta P^{0.5373} \quad \textit{for}\ 15''\textit{-diameter cyclone} \tag{1.57d}$$

$$Q = 260\Delta P^{0.52158} \quad \textit{for}\ 20''\textit{-diameter cyclone} \tag{1.57e}$$

$$Q = 440\Delta P^{0.5391} \quad \textit{for}\ 26''\textit{-diameter cyclone} \tag{1.57f}$$

$$Q = 840\Delta P^{0.4794} \quad \textit{for}\ 30''\textit{-diameter cyclone} \tag{1.57g}$$

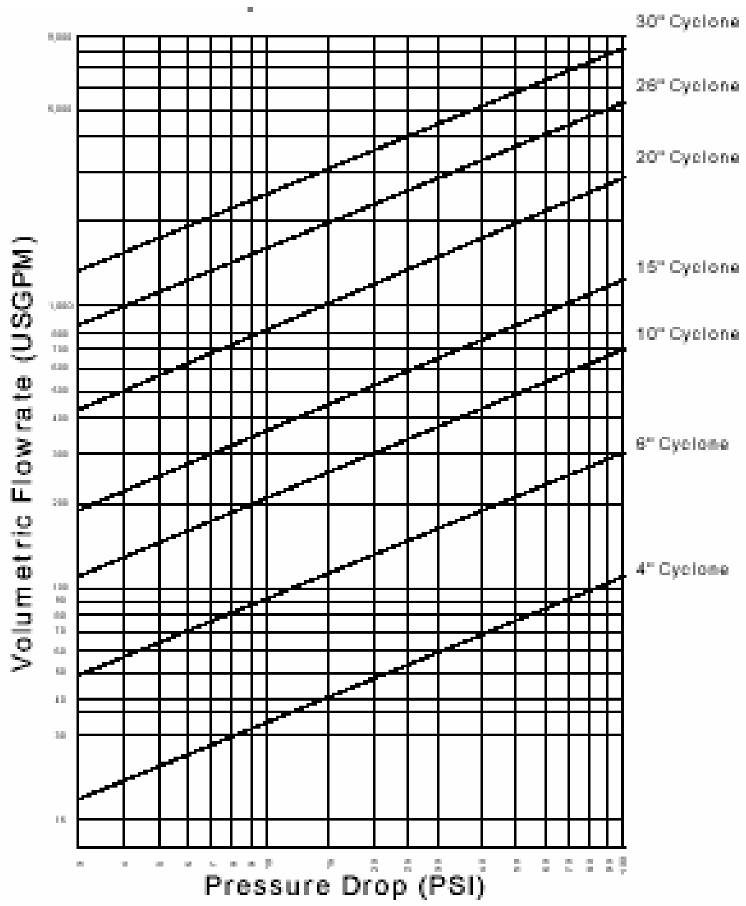

Figure 1.11 Pressure drop versus volumetric flow rate.

1.55 Design Based on Required Flow Rate and Cut Size by Optimization

The previous section dealt withy cyclone design when the operating static pressure drop was given. If the pressure drop is not known, then there are many possible solutions to a given design problem (Svarowsky, 1981). At a given flow rate, the required cut size, d50, can be achieved by using one or only a few large cyclones operated in parallel at a relatively large pressure drop or, alternatively, by using a great number of small units operated at a lower pressure drop. Lower pressure drop means lower running costs but higher capital charges. Therefore, there might be an optimum solution to this problem.

Gerrard and Liddle (1975) have described two alternative methods for selection of the cyclone diameter and the number of units to be used in parallel for a given duty. Both methods were based on the optimization of the operating costs and fixed charges per unit time and were linked to Rietema's optimum design proportions and the pressure drop relationship. Both alternatives assume a power-law relationship for the capital cost of an individual hydro cyclone and also that the pump and pumping costs together with the installation and maintenance charges are proportional to the capital cost of the cyclones.

The first alternative relies on knowledge of the desired cut size (as may be the case for classification duties) and it gives the optimum number of cyclones, their diameter, the expected pressure drop, the inlet Reynolds number (if required) and also the minimum value of the negative profits (operating costs plus fixed charges).

The calculation utilizes different cyclone numbers, which enables the user see how the behavior of the resulting parameters varies around the optimum.

The second method includes the cut size in the optimization (as is the case in separation applications) by adding the cost of the unseparated solids (lost product) to the running costs. The particle size distribution of the feed solids is assumed to follow the Rosin-Rammler equation with the exponent equal to unity, and an analytical equation used for the reduced grade efficiency curve. The calculation yields an optimum number of cyclones (to be rounded off to the nearest integer) as well as the optimum cut size. The evaluation is iterative.

2/ Design Equations

2.1 INTRODUCTION

The different designs mentioned in Chapter 1 were thoroughly examined and used to formulate the equations that were programmed in the software. These equations are presented in this Chapter. For ease of cross referencing, the equations numbers used in Chapter 1 are retained where applicable and the computational algorithm is presented.

2.2 DESIGN EQUATIONS

Cut Size d_{50} models

A total of 8 models for predicting cyclone cut point, d_{50}, were implemented in the software. These models are summarized in Table 2.1 and the various equations are enumerated.

Table 2.1 Models for d_{50}

Model Description
Dahstrom (1954): Small Diameter, Dilute Slurries
Bradley 1960 Empirical Model - Preliminary Calculations
General Model: Scale Up At Low Feed Concentrations
Generalised Model With Effects of Underflow To Throughput Ratio
Plitt (1976); Large Diameter, High Feed Concentration
Mular & Jull (1978); Preliminary Design Purposes. Large Diameter, High Feed Concentration
Krebbs Engineers - Correction for Feed Concentration, Density and Pressure Drop
Massarani 1997: Correction for Feed Concentration and Underflow to Throughput ratio

Dahstrom (1954): Small Diameter, Dilute Slurries

$$d_{50} = \left(\frac{13.7(D_o D_c)^{0.68}}{Q^{0.53}(\rho_S - \rho_L)^{0.5}} \right) \tag{1.37}$$

where Q is in m^3/h, d_{50} in microns (μm), D_c, D_o in cm, S and L are densities of solid and liquid, respectively, in g/cm^3.

Bradley 1965 Emperical Model - Preliminary Calculations

$$d_{50} = 4.5\left(\frac{D_c^3 \mu}{L^{1.2}(\rho_s - \rho_l)} \right) \tag{2.1}$$

where L is in litres/min, d_{50} in microns (μm), D_c in cm, ρ_s and ρ_l are densities of solid and liquid, respectively, in g/cm^3, μ is liquid viscosity in centipoises (mNs/m^2)

Generalised Model: Scale Up At Low Feed Concentrations

$$Eu = \frac{2\Delta P}{\rho v^2} \tag{1.32}$$

$$Stk_{50} = \frac{d_{50}^2(\rho_s - \rho_l)v}{18\mu D_c} \tag{1.33}$$

$$\mathrm{Re} = \frac{\rho v D_c}{\mu}, \tag{1.6}$$

$$v = \frac{4Q}{\pi D_c^2} \tag{1.15}$$

$$Stk_{50} Eu = Cons\tan t \tag{1.29}$$

$$Eu = K_p \mathrm{Re}^{n_p} \tag{1.30}$$

$$d_{50}^2 = Stk_{50}.Eu \frac{36\rho\mu Q}{\pi \Delta\rho \Delta P D_c} \tag{1.36}$$

$$D_c^{4+n_p} = \left(\frac{4Q}{\pi} \right)^{2+n_p} \left(\frac{\rho}{\mu} \right)^{n_p} \frac{K_p \rho}{2\Delta P} \tag{1.35}$$

Units in S.I. units

Generalised Model With Effects of Underflow To Throughput Ratio

For C<10%

$$Eu = 71(\mathrm{Re})^{-0.116} \left({}^{D_i}\!/\!{}_{D_c} \right)^{-1.3} Exp(2.12C) \tag{1.44}$$

$$R_f = 1218 \left({}^{D_u}\!/\!{}_{D_c} \right)^{-4.75} Eu^{-0.30} \tag{1.45}$$

$$Stk_{50}(r)Eu = 0.047\left[\ln\left(\frac{1}{R_f}\right)\right]^{0.74} Exp(8.96C) \qquad (1.43)$$

For C > 10%

$$Eu = 1686\left(\mathrm{Re}^*\right)^{-0.035} Exp(-3.39C) \qquad (1.47)$$

$$R_f = 32.8\left(\frac{D_u}{D_c}\right)^{1.53}\left(\mathrm{Re}^*\right)^{-0.34} Exp(3.70C) \qquad (1.48)$$

$$Stk_{50}^*(r)Eu = 0.006\left[\ln\left(\frac{1}{R_f}\right)\right]^{2.37} Exp(6.84C) \qquad (1.46)$$

where C is the volumetric concentration of solids in feed slurry, Re is Reynolds No, Eu is Euler No, Stk_{50} is Stokes No. Units in S.I. units

$$d_{50}^2 = Stk_{50}^*.Eu\frac{36\rho\mu Q}{\pi\Delta\rho\Delta PD_c} \qquad (1.36)$$

$$D_c^{4+n_p} = \left(\frac{4Q}{\pi}\right)^{2+n_p}\left(\frac{\rho}{\mu}\right)^{n_p}\frac{K_p\rho}{2\Delta P} \qquad (1.35)$$

Plitt (1976); Large Diameter, High Feed Concentration

$$d_{50} = \frac{14.8D_c^{0.68}D_i^{0.6}D_o^{1.21}Exp(0.063V)}{D_u^{0.71}h^{0.38}Q^{0.45}(\rho_S-\rho_L)^{0.5}} \qquad (1.38)$$

$$Q = \frac{0.021P^{0.56}D_c^{0.21}D_i^{0.53}h^{0.16}\left(D_u^2+D_o^2\right)^{0.49}}{Exp(0.0031V)} \qquad (1.39)$$

where P is the pressure drop in kpa, V is volumetrice percent of solids in feed, Q is in m^3/h, d_{50} in microns (μm), D_c, D_i, D_o, h in cm, S and L are densities of solid and liquid, respectively, in g/cm^3.

Mullar & Jull (1978); Preliminary Design Purposes. Large Diameter, High Feed Concentration

$$d_{50} = \frac{0.77D_c^{1.875}Exp(-0.301+0.0945V-0.00356V^2+0.0000684V^3)}{Q^{0.6}(S-1)^{0.5}} \qquad (1.40)$$

$$Q = 9.4\times10^{-3}\sqrt{\Delta P}D_c^2 \qquad (1.41)$$

where ΔP is the pressure drop in kpa, V is volumetrice percent of solids in feed, Q is in m^3/h, d_{50} in microns (μm), D_c, D_i, D_o, h in cm, S and L are densities of solid and liquid, respectively, in g/cm^3.

Krebbs Engineers - Correction for Feed Concentration, Density and Pressure Drop

$$d_{50} = 5.27D^{0.66}C_1C_2C_3 \tag{1.49}$$

$$C_1 = \left(\frac{53-V}{53}\right)^{1.43} \tag{1.52}$$

$$C_2 = \left(\frac{1.65}{\rho_s - \rho_l}\right)^{0.5} \tag{1.53}$$

$$C_3 = 1.91\Delta P^{-0.28} \tag{1.54}$$

where P is the pressure drop in psi, V is volumetrice percent of solids in feed, Q is in m^3/h, d_{50} in microns (μm), D is inches, ρ_s and ρ_l are densities of solid and liquid, respectively, in g/cm^3.

Massarani 1997: Correction for Feed Concentration and Underflow to Throughput ratio

$$\frac{d'_{50}}{D_c} = K\left(\frac{\mu D_c}{Q(\rho_s - \rho_l)}\right)^{0.5} F(R_L)G(C_v) \tag{1.18}$$

$$F(R_L) = \frac{1}{1+1.73R_L} \tag{1.19}$$

$$G(C_v) = Exp(4.5C_v) \tag{1.20}$$

$$R_L = B\left(\frac{D_u}{D_c}\right)^C \tag{1.21}$$

where C_v is volumetrice percent of solids in feed, Q is in m^3/s, d_{50} in microns (μm), D_c, in m, ρ_s and ρ_l are densities of solid and liquid, respectively, in kg/m^3, μ is liquid viscosity in Ns/m^2, K, B and C are constants.

Efficiency (Recovery) Curve

Three (3) efficiency curve models were implemented in the software to predict the overall efficiency of the cyclone. These models are given below.

Yoshioka & Hotta (1955)

$$y = \frac{e^{\alpha x} - 1}{e^{\alpha x} + e^{\alpha} - 2} \tag{2.2}$$

where $x = d/d_{50}$

Gerrard & Liddle (1976)

$$y = 1 - \left[1 + 2.142x + 3.463x^2 - 2.508x^3\right]Exp(-2.142x) \tag{2.3}$$

where x=d/d_{50}

Lynch et. al. (1974)

$$y = 1 - Exp\left[-(x - 0.115)^3\right] \tag{2.4}$$

where x=d/d_{50}

Particle Distribution in feed

Establishing the overall efficiency of a cyclone requires knowledge of the distribution of the solids particle sizes in both the feed and the underflow, as well as knowledge of the maximum particle size. The program makes provision for the following 3 situations;

- Normal distribution of particles in feed,
- Even distribution of particles in feed and
- Experimental data of particle distribution in feed.

If normal distribution is assumed the user is required to supply the mean and standard deviation of the particles in the feed. The maximum particle size is assumed to be twice the mean. If even distribution of particles is assumed the user is required to supply the maximum particle size. With experimental data on distribution, the user is required to enter the data in tabulated form and the maximum particle size is extracted automatically.

Imperfection

$$I = \frac{d_{75} - d_{25}}{2d_{50}} \tag{1.1}$$

Overall Efficiency

$$E_T = \frac{W_{su}}{W_s} \tag{1.6}$$

Reduced Overall Efficiency

$$E_Y^{'} = \frac{E_T - R_f}{1 - R_f} \tag{1.2}$$

Percentage (%) Solids by weight in feed

$$X = 100\frac{\rho_s(\rho_f - \rho_w)}{\rho_f(\rho_s - \rho_w)} \tag{1.7}$$

Percentage (%) Solids by volume in feed

$$V = 100\frac{(\rho_f - \rho_w)}{(\rho_s - \rho_w)} = X\frac{\rho_f}{\rho_s} \tag{1.8}$$

Cone Angle

We derived a model relating the cone angle of a cyclone to the conical length, cyclone diameter and apex diameter.

$$Tan\left(\frac{\theta}{2}\right) = \frac{D_c - D_u}{2L_c} = \frac{1-\alpha_u}{2\alpha_{L_c}} \tag{1.9}$$

where θ is the cone angle in degrees, D_c is the cyclone diameter, D_u is the apex diameter, L_c is the conical length of the cyclone, α_u and α_{Lc} are the ratios of the apex diameter and conical length to the cyclone diameter.

2.3 DESIGN PARAMETERS

Given the cyclone geometry and the operating conditions (operating temperature, operating pressure, feed density, and viscosities), there are 4 design parameters that can be specified for a design, namely:

1. Feed Rate, Q, in m^3/h
2. Cyclone Diameter, Dc, in cm
3. Set Cut Size, d_{50}, in microns
4. Pressure Drop, ΔP in Kpa

Depending on the problem and available information, the program provides for 6 possible combinations of these design parameters (see Table 2.2).

Table 2.2 Possible design parameters combinations

S/N	Design Parameters	Values to derive from parameters
1	Feed Rate (Q) and Pressure Drop (ΔP)	Dc and d_{50}
2	Feed Rate (Q) and Cyclone Diameter (Dc)	ΔP and d_{50}
3	Pressure Drop (ΔP) and Cyclone Diameter (Dc)	d_{50} and Q
4	Cut Size (d_{50}) and Feed Rate (Q)	Dc, ΔP and Ncyclones
5	Cut Size (d_{50})and Pressure Drop (ΔP)	Dc and Q
6	Cut Size (d_{50})and Pressure Drop (ΔP) and Feed Rate (Q)	Dc, Q and Ncyclones

Ncyclones = number of cyclones required for duty

3/ Computer Aided Design of Hydrocyclone

3.1 INTRODUCTION

Extensive literature review on gydrocyclone design was carried out and the various design methodologies, models and approaches have been thoroughly examined in previous sections. The established designs were analysed and collated to formulate a comprehensive design sequence for computational implementation. We employed a modular approach to developing the software programs, providing room for extension to other unit operations.

3.2 SOFTWARE DESIGN COMPUTATIONAL ALGORITHM

1. Select Unit Operation: i.e. Select Cyclone>Hydrocyclone
2. Select Hydrocyclone Geometry
3. Select d_{50} model
4. Input solids density
5. Input liquid density
6. Input Feed pulp density OR Weight % Solids in Feed OR Volumetric % Solids in Feed
7. Input liquid viscosity
8. Input feed pulp viscosity
9. Input Maximum Particle Size, if available
10. Input Minimum Particle Size, if available
11. Select Particle Distribution Type
    a. Enter the mean and standard deviation, if Normal Distribution assumed
    b. Enter maximum particle diameter, if even distribution is assumed
    c. Enter particle distribution data, if measured data is available
12. Select Efficiency Model
13. Select the combination of the design parameters available from

  a. Feed Rate, Q
  b. Cyclone Diameter, Dc
  c. Set Cut Size, d_{50}
  d. Pressure Drop, ΔP

Design Parameters Combinations
Feed Rate (Q) and Pressure Drop (ΔP)
Feed Rate (Q) and Cyclone Diameter (Dc)
Pressure Drop (ΔP) and Cyclone Diameter (Dc)
Cut Size (d_{50}) and Feed Rate (Q)
Cut Size (d_{50})and Pressure Drop (ΔP)
Cut Size (d_{50})and Pressure Drop (ΔP) and Feed Rate (Q)

14. Enter the design parameters based on selection made in 14. This computes the cyclone dimensions and the other design parameters not in the combination
    a. Click Compute Design
    b. Compute %Solids by wt in feed
    c. Compute %Solids by vol in feed
    d. Compute Underflow to throughput ratio (water)
    e. Compute Dilution Ratio of feed
    f. Compute Water flow rate in feed
    g. Compute Solids flow rate in feed
    h. Generate Reduced Efficiency Curve
    i. Compute Overall Efficiency
    j. Compute Reduced Overall Efficiency
    k. Compute Imperfection
    l. Compute Underflow Density
    m. Compute Overflow Density
    n. Compute Overall Mass Balance
    o. Compute Overall Water Balance
    p. Compute Overall Solids Balance
    q. Check Mass Balance
    r. Display Reduced efficiency curve when prompted.
15. Display Outputs.
16. Print results to a file for future reference

4/ Software Validation

4.1 INTRODUCTION

The software generates the cyclone dimensions from a selected geometric configuration and an acceptable combination of at least 2 or 3 parameters from the 4 design inputs that required by the software.

The efficiency model, cut point model, solids distribution in feed, feed, liquid and solids densities, feed and liquids viscosities are all inputs required by the program to compute the various outputs such as overall cyclone efficiency, mass balances and cyclone dimensions. The software generates the cyclone dimensions once enough parameters have been entered to compute the cyclone diameter, Dc.

Another significant feature of the program is that it accepts inputs in any order. The user is not forced through a particular sequence, but must completely specify all the required inputs. The program incorporates a management utility that coordinates the input information in the background and only computes required data when information is complete.

Facilities exists to capture, in the database, new geometric configurations and efficiency models as well as update existing configurations. A list of densities of known substances is also available in the database, which can be extended as new information are gathered.

Essentially the overall procedure to use the software for design is as follows

- Select the geometric configuration
- Specify the efficiency model
- Specify design parameters
- Calculate design variables
- Check the design outputs meet constraints
- Save the solution in a file for future reference

Important Considerations

Using the software, the solids particles distribution in feed can be defined. However, in many cases feed solids distribution and size analysis are not available and the program provides for this situation offering 2 assumptions.

- Normal Distribution of particles in feed
- Even distribution of particles in feed

If even distribution of particles in the feed is assumed the program assumes the maximum particle size in 3 times the d_{50} in order to be able to compute the reduced efficiency curve and obtain a values for the overall efficiency. However, you must take into account that the overall efficiency computed this way is affected by the value of the maximum particle size and so caution must be exercised when using this assumption. Normal distribution of particles is a more reliable assumption, and when used, the program computes efficiency assuming maximum particles size is twice the mean. If feed screen analysis is available, the program automatically determines the maximum particle size to compute efficiency.

4.2 CASE STUDIES

The software computes the cyclone dimensions once enough information have been specified. To assess the correctness or otherwise of the output generated by the software some examples from literature were used. Four problems examined as cases studies are presented in this monograph:

Problem 5.1 - Specify Feed Rate and Pressure Drop

Rietema Design Configuration
Q =18m^3/h
ΔP =100kPa
ρ_s =3000 kg/m^3
ρ_l =1000 kg/m^3
%V =1% volumetric percent in feed (i.e. ρ_f=1020 kg/m^3)
μ_l =0.001Ns/m^2
Mean particle size = 15μm, Standard Deviation = 3 μm
d_{50} model: Scale Up Model for Low Feed Concentrations
Efficiency Model: Yoshioka & Hotta

Problem 5.2 - Specify Feed Rate and Pressure Drop

Bradley Design Configuration
Q =$18m^3/h$
ΔP =100kPa
ρ_s =3000 kg/m^3
ρ_l =1000 kg/m^3
%V =1% volumetric percent in feed (i.e. ρ_f=1020 kg/m^3)
μ_l =$0.001Ns/m^2$
Mean particle size = 15 µm, Standard Deviation = 3 µm
d_{50} model: Scale Up Model for Low Feed Concentrations
Efficiency Model: Yoshioka & Hotta

Problem 3 - Specify cut point, pressure drop, and duty

Rietema Design Configuration
d_{50} =8 µm
ΔP =305.24kPa
Q =$30m^3/h$
ρ_s =2600 kg/m^3
ρ_l =1000 kg/m^3
%V =1% volumetric percent in feed (i.e. ρ_f=1016 kg/m^3)
μ_l =$0.001Ns/m^2$
Mean particle size = 15 µm
Standard Deviation = 3 µm
d_{50} model: Scale Up Model for Low Feed Concentrations
Efficiency Model: Yoshioka & Hotta

Problem 4
Problem 3 was repeated using six other d_{50} models implemented in the program.

Problem 5 - Medium/High Feed Concentration: Specify cut point, pressure drop, and duty, using seven d_{50} models.

Rietema Design Configuration
d_{50} =74 µm, ΔP =82.74kPa, Q =$1024m^3/h$
ρ_s =3700 kg/m^3, ρ_l =1000 kg/m^3
%V =21.67% volumetric percent in feed (i.e. ρ_f=1574 kg/m^3)
μ_l =$0.001Ns/m^2$
Mean particle size = 110 µm, Standard Deviation = 3 µm
d_{50} model: Mullar and Jull
Efficiency Model Used in Software: Yoshioka & Hotta

Reference: Mineral Processing Technology; Chapter 9, P364-366

The results obtained when the software solved Problems 1 to 3 are summarized in Table 5.1. while that for Problem 4 is shown in Table 5.2. The details are given in Appendix A.

Table 5.1 Solution to Problems 1 to 3

	Problem 1		Problem 2		Problem 3	
	Calculated	Literature	Calculated	Literature	Calculated	Literature
D_c; in cm	12.971	12.970	22.330	22.330	8.838	9.100
d_{50}; in μm	11.617	11.620	11.939	11.930		
Reduced Efficiency %, E'_T	72.194		71.556		96.283	
No of cyclones	1	1	1	1	2	2
Qmax	15.816		46.861		14.204	

Table 5.2 Solution to Problem 4

d_{50} Model	D_c; in cm	No of Cyclones	Reduced Efficiency %, E'_T	**Qmax, m^3/h**
Problem 3 (Base case)	**8.838**	**2**	**96.283**	14.204
Bradley 1960	15.924	1	96.283	25.408
Dahlstrom	3.094	19	96.399	1.572
Scale Up For All Concentrations	7.567	3	96.271	10.671
Plitt	2.444	42	96.049	0.722
Mullar & Jull	12.445	1	96.303	25.436
Krebs	8.394	3	96.283	13.470

Table 5.1 shows that the results agree very well with literature as regards the outputs, that is, cut size and cyclone diameter, thus validating the functional integrity and correctness of the software.

Table 5.2 shows that the results for Krebs and Massarani Scale-up Model for all concentrations agree with the base case. Bradley, Dahlstrom, Plitt and Mullar & Jull models are not in agreement. This confirms that Krebbs and Massrani models are valid for the low feed concentration (1% vol) used while the other models are not. The values of Q_{max} for Dahlstrom and Plitt also appear rather too low which again makes the two models odd for the problem. It may be pointed out that Plitt model is based on empirical results for large diameter cyclones and high feed concentrations. Although the Dahlstrom model is for small diameter cyclones and dilute slurries but the exact ranges are not specified.

The result obtained when Problem 5 was solved is shown in Table 5.3 using different d_{50} models. In the literature the problem was solved with the Mullar and Jull model and Table 5.3 confirms that the software output for the Mullar and Jull model agrees very well with literature. The Plitt and Bradley models also agree fairly well with literature values although the configuration would be more expensive (larger diameter in the case of Bradley and more cyclones in the case of Plitt). As expected the Dahlstrom and Scale Up for low concentrations models gave results that are not feasible since these models are for dilute slurries. The Krebs model gave a slightly higher diameter than the literature value.

Table 5.3 Solution to Problem 5

d_{50} Model	D_c; in cm	No Cyclones	Reduced Efficiency %, E'_T	**Qmax, m^3/h**
Mullar & Jull (Literature)	66	3	-	372.5
Software				
Mullar & Jull	66.223	3	86.428	374.981
Plitt	64.446	7	85.911	153.437
Bradley 1960	116.223	3	86.119	374.565
Dahlstrom	3880.678	1	87.442	1287658.95
Scale Up For Low Concs	1320.926	1	87.192	83086.57
Scale Up For All Concs	340.716	1	86.91	6845.96
Krebs	71.611	1	87.192	4504.34

List Of Notations

a	Inlet height, m
a_o	Inlet height in ratio of Dc
A	Surface area of cyclone exposed to gas, m^2
Ac	Cross sectional area of cyclone, m^2
b	Inlet width, m
b_o	Inlet width, in ratio of Dc
B	Dust outlet diameter, m
B_o	Dust outlet diameter, in ratio of Dc
C_{fixed}	Fixed cost of cyclone
C_{power}	Power cost of running cyclone
d	Particle diameter, m
d_n	Diameter of core where vortex turns, m
d_{pc}, d_{50}	Cut size (critical diameter at 50% efficiency), microns (d_{50})
$d_{p,critical}$	Critical particle size diameter, microns
d_1	Original particle size, microns
d_2	New particle size, microns
Dc	Cyclone diameter, m (D)
De	Gas outlet diameter, m
De_o	Gas outlet diameter, in ratio of Dc
Di	Feed Inlet Diameter, cm
Do	Overflow Diameter, cm
Du	Apex/Spigot Diameter, cm
e	$\$/m^j$
E_{d1}	Original efficiency at particle size $d_{1,}$ %
E_{d2}	New efficiency at particle side $d_{2,}$ %
E_T	Total collection efficiency, %
f	Investment factor
G	Friction constant (0.005 for gas cyclones)
GSD	Geometric standard deviation of the inlet particle size distribution.
h	Cylindrical height of cyclone, m
h_o	Cylindrical height of cyclone, in ratio of Dc

H	Total height of cyclone (h+Zc), m
H’	Time worked by cyclone per year, seconds/year
H_o	Total height of cyclone (h+Zc), in ratio of Dc
Hpd	Hopper diameter, m
Hpd_o	Hopper diameter, in ratio of Dc
i	Imperfection, dimensionless
j	Capital cost exponent of cyclone diameter
K	Cyclone Configuration factor
K_c	Cyclone Volume Constant
l	Cyclone natural length, m
L	Cyclone Length, cm
Lcyl	Cylindrical length of cyclone, cm
Lc	Conical length of cyclone, cm
Lvf	Vortex Finder length, cm
m_j	Mass of particle in j^{th} size range
M	Total mass of particle, kg
MMD	Mass median diameter of the inlet particle size distribution
n	Vortex exponent
N	No of cyclones in parallel
N_H	No of inlet velocity heads
N_o	Optimum number of cyclones in parallel
N_t	No of effective turns made by gas stream
P	Operating pressure, N/m^2
P(d)	Particle collection probability at any point (P(r,z)
PM	Particulate matters
P_1	Absolute pressure at cyclone inlet, N/m^2
P_2	Absolute pressure at cyclone outlet, N/m^2
Q	Feed rate, m^3/s
Q_1	Original cyclone flow rate, m^3/s
Q_2	New cyclone flow rate, m^3/s
r	Radial point
S	Gas outlet length, m
SCC	Sharp-cut cyclone
S_f	Separation factor
S_o	Gas outlet length, in ratio of Dc
T	Operating temperature, ^oC or ^oF
V_H	Volume of gas below the gas exit duct excluding the core, m^3
V_i, v_i	Inlet velocity, m/s
V_s, v_s	Saltation velocity, m/s
VSCC	Very sharp-cut cyclone
V_o, v_o	Outlet velocity, m/s
W	Dust loading, grain/acf $[g/m^3]$

Y	No of years over which depreciation occurs
z	Axial point
z_o	Vortex length also known as the cyclone effective length, m
Zc	Conical height of cyclone, m
Zc_o	Conical height of cyclone, in ratio of Dc
α_1	Length factor for cylindrical portion of cyclone
α_2	Length factor for vertical cone portion of cyclone
θ	Cone Angle in degrees
ΔP	Pressure drop, N/m^2
ΔP_1	Original cyclone pressure drop, N/m^2
ΔP_2	New cyclone pressure drop, N/m^2
ρ_f	Fluid density, kg/m^3
ρ_{fp}	feed pulp density, kg/m^3
ρ_s	solids density, kg/m^3
ρ_g	Gas density, kg/m^3
ρ_{g1}	Original gas density, kg/m^3
ρ_{g2}	New gas density, kg/m^3
ρ_p	Particle density, kg/m^3
ρ_w	liquid density, kg/m^3
μ_f	fluid viscosity Ns/m^2
μ_w	liquid viscosity Ns/m^2
η	Collection efficiency, %
η'	Reduced Overall efficiency, %
η_j	Collection efficiency of jth particle size range, %
η_o	Overall collection efficiency, %
φ	Friction factor
ΔP	Pressure Drop, Kpa
Q	Feed rate, m^3/h

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